AFRL-ML-WP-TP-2007-409

PROBABILISTIC ANALYSIS FOR THE MECHANICAL PROPERTIES OF CROSS-PLY FIBER-REINFORCED COMPOSITE LAMINATE (POSTPRINT)

Shui-Nan Chuang

MARCH 2006

Approved for public release; distribution unlimited.

STINFO COPY

MATERIALS AND MANUFACTURING DIRECTORATE
AIR FORCE RESEARCH LABORATORY
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7750

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
March 2006	Conference Paper Postprint	

4. TITLE AND SUBTITLE
PROBABILISTIC ANALYSIS FOR THE MECHANICAL PROPERTIES OF CROSS-PLY FIBER-REINFORCED COMPOSITE LAMINATE (POSTPRINT)

5a. CONTRACT NUMBER In-house
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER 62102F

6. AUTHOR(S)
Shui-Nan Chuang

5d. PROJECT NUMBER 4347
5e. TASK NUMBER RG
5f. WORK UNIT NUMBER M03R1000

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Structural Materials Branch (AFRL/MLBC)
Nonmetallic Materials Division
Materials and Manufacturing Directorate
Air Force Research Laboratory, Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7750

8. PERFORMING ORGANIZATION REPORT NUMBER
AFRL-ML-WP-TP-2007-409

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Materials and Manufacturing Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson AFB, OH 45433-7750

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL-ML-WP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-ML-WP-TP-2007-409

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
Conference paper published in the Proceedings of the 2006 ASME International Mechanical Engineering Congress, published by ASME.
 PAO Case Number: AFRL/WS 06-1064, 24 Apr 2006. Paper contains color content.

14. ABSTRACT
A probabilistic micromechanics model has been developed for the unidirectional fiber-reinforced composite material design screening. This model provides a design-screening tool to help material producers eliminate the unnecessary time-consuming and costly material fabrications and to reduce the numbers of testing to a minimum, but enough to verify the model prediction. Consequently, it provides a means to accelerate the insertion of materials into AF productions. To include the material design in the structural design process, we extended the probabilistic analysis to predict the probability density functions for the off-axis transformed reduced stiffness, Young's modulus, transverse Young's modulus, shear modulus and Poisson's ratio. In this report we will focus on the probabilistic analysis of symmetric and anti-symmetric regular cross-ply laminates of IM-7/520-4 fiber-reinforced composite with odd-number plies parallel to and even-number plies perpendicular to the laminate principal axes.

15. SUBJECT TERMS
Probabilistic analysis, classical lamination theory, laminated composite

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
Unclassified	Unclassified	Unclassified	SAR	20	Shui-Nan Chuang
					19b. TELEPHONE NUMBER *(Include Area Code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

PROBABILISTIC ANALYSIS FOR THE MECHANICAL PROPERTIES OF CROSS-PLY FIBER-REINFORCED COMPOSITE LAMINATE

Shui-Nan Chuang
Materials and manufacturing Directorate, Air Force Research Laboratory
AFRL/MLBC, Wright-Patterson AFB, OH 45433-7750
Phone: (937) 656-9102, E-mail: shui.chuang@wpafb.af.mil

ABSTRACT

A probabilistic micromechanics model had been developed for the unidirectional fiber-reinforced composite material design screening. In which, we used the predicted mechanical properties of IM-7 carbon fiber from the existing IM-7/5250-4 composite material system together with the observed 977-3 matrix mechanical properties to predict the probability density functions for the mechanical properties of IM-7/977-3 unidirectional composite. To include the material design in the structural design process, we had extended the probabilistic analysis to predict the probability density functions for the off-axis mechanical properties. The angle-ply and cross-ply laminates have been used extensively in aerospace structural designs. It is logical to extend the probabilistic analysis to predict the probability density functions for the mechanical properties of the laminated composite. We had provided the probabilistic analysis for a symmetric regular angle-ply laminate of IM-7/5250-4 composite laminate.

In this report, we will focus on the probabilistic analysis of symmetric and anti-symmetric regular cross-ply laminates of IM-7/5250-4 fiber-reinforced composite with odd-number plies parallel to and even-number plies perpendicular to the laminate principal axes.

These probabilistic micromechanics models provide a design-screening tool to help material producers to eliminate the unnecessary time-consuming and costly material fabrications and to reduce the numbers of testing to a minimum but enough to verify the model prediction. They also provide a structural analysis tool to help the structural designer to manage the structural and material uncertainties during the structural design process. And consequently, it provides a means to accelerate the insertion of materials into AF productions.

KEYWORDS: Probabilistic Analysis, Laminated Composite

INTRODUCTION

Insertion of advanced composite materials into Air Force (AF) productions requires extensive testing of mechanical and other physical properties at the coupon, element, subcomponent, and component levels. The material development becomes a time-consuming and costly task, and new material insertion into productions is extremely difficult, typically taking 15 to 20 years. To accelerate the insertion of composite materials into AF productions at a much lower cost, a means must be established to preclude premature material fabrications and reduce material testing to a minimum but enough to manage the uncertainties in material development.

A probabilistic micromechanics model had been developed [1] for the unidirectional fiber-reinforced composite material design screening. In which, we used the predicted mechanical properties of IM-7 carbon fiber from the existing IM-7/5250-4 composite material system together with the observed 977-3 matrix mechanical properties to predict the probability density functions for the mechanical properties of IM-7/977-3 unidirectional composite. This probabilistic micromechanics model provides a design-screening tool to help material producers to eliminate the unnecessary time-consuming and costly material fabrications and testing. And consequently, it provides a means to accelerate the insertion of materials into AF productions.

To include the material design in the structural design process, we had extended the probabilistic analysis [2] to predict the probability density functions for the off-axis transformed reduced stiffness, Young's modulus, transverse Young's modulus, shear modulus and Poisson's ratio. In which, the probabilistic analysis of the classical lamination theory were utilized to demonstrate the variation of the predicted probability density functions for these mechanical properties of IM-7/5250-4 unidirectional composite for a sequence of off-axis angles from the material principal direction.

The angle-ply and cross-ply laminates have been used extensively in aerospace structural designs. It is logical to extend the probabilistic analysis to predict the probability density functions for the extensional, coupling and bending stiffness, Young's modulus, transverse Young's modulus, shear modulus and Poisson's ratio of fiber-reinforced composite laminates. In which, the probabilistic analysis of the classical lamination theory are utilized to demonstrate the variations of the predicted probability density functions for these mechanical properties of the laminated composites with thickness, stacking sequence, and the angles off-axis from the laminate's principal direction. In [3], we had provided a probabilistic analysis on a symmetric regular angle-ply laminate. In this report, we will focus on the probabilistic analysis of symmetric and anti-symmetric regular cross-ply laminates of IM-7/5250-4 fiber-reinforced composite with odd-number plies parallel to and even-number plies perpendicular to the laminate principal axes.

The probabilistic micromechanics model for composite laminates provides a tool for performing preliminary material design analyses and screening as commonly used by the structural design engineers in the general derivation of probability of failure as depicted in Figure 1 [4]. The predictions by the probabilistic structural and material analysis models will provide the means for the structural designers to manage the structural and material uncertainties during the structural design process.

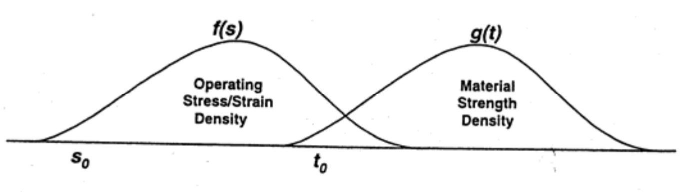

Figure 1. General Derivation of Probability of Failure (From Technical Report DOT/FAA/AR-95/17)

PROBABILISTIC ANALYSIS

The orthotropic unidirectional composite laminas [5, 6] satisfies

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \qquad (1)$$

where

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \qquad (2)$$

$$Q_{66} = G_{12}$$

(A) SYMMETRIC CROSS-PLY LAMINATE

For an odd number N-layered regular cross-ply laminate, which is symmetric in both geometry and material properties about the middle surface, with equal thickness t of orthotropic lamina and has an overall thickness of h=Nt, the resultant forces and moments per unit length satisfies the following stiffness equations

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} \qquad (3)$$

and

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix}\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix} \qquad (4)$$

in which the extensional stiffness A_{ij} and bending stiffness D_{ij} of the orthotropic fiber-reinforced cross-ply laminate are

$$\begin{Bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \end{Bmatrix} = h \begin{bmatrix} \frac{M}{1+M} & \frac{1}{1+M} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{1+M} & \frac{M}{1+M} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} = Nt \begin{bmatrix} \frac{M}{1+M} & \frac{1}{1+M} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{1+M} & \frac{M}{1+M} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} \tag{5}$$

and

$$\begin{Bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{66} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1-P & P & 0 \\ 0 & \nu_{12} & 0 \\ P & 1-P & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} = \frac{N^3 t^3}{12} \begin{bmatrix} 1-P & P & 0 \\ 0 & \nu_{12} & 0 \\ P & 1-P & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} \tag{6}$$

in which the cross-ply ratio M, the ratio of the total thickness of odd-numbered layers to the total thickness of even-numbered layers, is defined as

$$M = \frac{N+1}{N-1} \tag{7}$$

and

$$P = \frac{1}{(1+M)^3} + \frac{M(N-3)[M(N-1)+2(N+1)]}{(N^2-1)(1+M)^3} \tag{8}$$

And the mechanical properties of the orthotropic fiber-reinforced cross-ply laminate in the principal direction satisfies

$$\begin{Bmatrix} \frac{1}{E_x} \\ \frac{1}{E_y} \\ \frac{\nu_{xy}}{E_x} \\ \frac{1}{G_{xy}} \end{Bmatrix} = \begin{bmatrix} \frac{M}{1+M} & \frac{1}{1+M} & 0 \\ \frac{1}{1+M} & \frac{M}{1+M} & 0 \\ \nu_{12} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{1}{E_1} \\ \frac{1}{E_2} \\ \frac{1}{G_{12}} \end{Bmatrix} \tag{9}$$

(B) ANTI-SYMMETRIC CROSS-PLY LAMINATE

For an even number N-layered regular cross-ply laminate, which is anti-symmetric in both geometry and material properties about the middle surface, with equal thickness t of orthotropic lamina and has an overall thickness of h=Nt, the resultant forces and moments per unit length satisfies the following stiffness equations

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{10}$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{11}$$

in which the extensional stiffness A_{ij}, coupling stiffness B_{ij}, and bending stiffness D_{ij} of the orthotropic fiber-reinforced cross-ply laminate are

$$\begin{Bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \end{Bmatrix} = h \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} = Nt \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} B_{11} \\ B_{22} \end{Bmatrix} = -\frac{h^2}{2N} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \end{Bmatrix} = -\frac{Nt^2}{2} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \end{Bmatrix} \tag{13}$$

and

$$\begin{Bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{66} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} = \frac{N^3t^3}{12} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \nu_{12} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} \\ \frac{E_2}{1-\nu_{12}\nu_{21}} \\ G_{12} \end{Bmatrix} \tag{14}$$

And the mechanical properties of the orthotropic fiber-reinforced cross-ply laminate in the principal direction satisfies

$$\begin{Bmatrix} \frac{1}{E_x} \\ \frac{1}{E_y} \\ \frac{\nu_{xy}}{E_x} \\ \frac{1}{G_{xy}} \end{Bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \nu_{12} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{1}{E_1} \\ \frac{1}{E_2} \\ \frac{1}{G_{12}} \end{Bmatrix} \tag{15}$$

With the probability density functions (PDFs) $f_{E_1}(E_1), f_{E_2}(E_2), f_{G_{12}}(G_{12})$ and deterministic value ν_{12} obtained from unidirectional composite analysis, the PDFs for random variables $X_{ij} = A_{ij}$ in Equations (5 and 12) and $X_{ij} = D_{ij}$ in Equations (6 and 14) in the format of $X_{ij} = a_{ij}E_1 + b_{ij}E_2 + c_{ij}G_{12}$ for ij = 11, 12, 22, 66, and $X_{ij} = B_{ij}$ in Equation (13) for ij = 11 and 22 respectively are

$$f_{X_{ij}}(X_{ij}) = \frac{1}{|a_{ij}|}\int_0^\infty\int_0^\infty f_{E_1}\left(\frac{X_{ij} - b_{ij}E_2 - c_{ij}G_{12}}{a_{ij}}\right) f_{E_2}(E_2)dE_2\, f_{G_{12}}(G_{12})dG_{12} \tag{16}$$

based on the probability and random variable analysis [7].

The PDFs for random variables $X_i = E_x, E_y \, or \, G_{xy}$ in the format of $\frac{1}{X_i} = \frac{a_i}{E_1} + \frac{b_i}{E_2} + \frac{c_i}{G_{12}}$ for i = 1, 2, 4 respectively in Equations (9 and 15) are

$$f_{X_i}(X_i) = \frac{|a_i|}{X_i^2}\int_0^\infty\int_0^\infty \frac{1}{\left(\frac{1}{X_i} - \frac{b_i}{E_2} - \frac{c_i}{G_{12}}\right)^2} f_{E_1}\left(\frac{a_i}{\frac{1}{X_i} - \frac{b_i}{E_2} - \frac{c_i}{G_{12}}}\right) f_{E_2}(E_2)dE_2\, f_{G_{12}}(G_{12})dG_{12} \tag{17}$$

The PDF for random variable ν_{xy} in Equation (9) is

$$f_{\nu_{xy}}(\nu_{xy}) = \int_0^\infty f_{\frac{\nu_{xy}}{E_x}}\left(\frac{\nu_{xy}}{E_x}\right) f_{E_x}(E_x)\left|\frac{1}{E_x}\right| dE_x \tag{18}$$

where

$$f_{E_x}(E_x) = \frac{|a_1|}{E_x^2}\int_0^\infty\int_0^\infty \frac{1}{\left(\frac{1}{E_x} - \frac{b_1}{E_2} - \frac{c_1}{G_{12}}\right)^2} f_{E_1}\left(\frac{a_1}{\frac{1}{E_x} - \frac{b_1}{E_2} - \frac{c_1}{G_{12}}}\right) f_{E_2}(E_2)dE_2\, f_{G_{12}}(G_{12})dG_{12} \tag{19}$$

and

$$f_{\frac{\nu_{xy}}{E_x}}\left(\frac{\nu_{xy}}{E_x}\right) = |a_3|\int_0^\infty\int_0^\infty \frac{1}{\left(\frac{\nu_{xy}}{E_x} - \frac{b_3}{E_2} - \frac{c_3}{G_{12}}\right)^2} f_{E_1}\left(\frac{a_3}{\frac{\nu_{xy}}{E_x} - \frac{b_3}{E_2} - \frac{c_3}{G_{12}}}\right) f_{E_2}(E_2)dE_2\, f_{G_{12}}(G_{12})dG_{12} \tag{20}$$

OBSERVED MECHANICAL PROPERTIES

To apply and demonstrate the probabilistic model developed in Section 2 for predicting the probability density functions for the extensional stiffness A_{ij}, coupling stiffness B_{ij}, bending stiffness D_{ij}, Young's modulus E_x, transverse Young's modulus E_y, shear modulus G_{xy}, and Poisson's ratio ν_{xy} for the regular angle-ply laminate with IM-7/5250-4 unidirectional fiber-reinforced orthotropic layers, an AFRL/MLBC in-house test program was established to measure the mechanical properties E_1, E_2, ν_{12} and G_{12} of IM-7/5250-4 unidirectional fiber reinforced composite.

The longitudinal unidirectional composite specimens were prepared with the configuration of eight-ply rectangular test specimen that was 0.5 inch wide and 10 inches long, and were tested in accordance with ASTM test procedure D3039. There are 63 specimens, with V_f = 0.63, tested at room temperature. A FORTRAN computer program was used to detect the outliers of the test results [8], and to calculate the frequencies of the histograms for the remaining 62 specimens. The test data were presented in the Appendix B of AFRL-ML-WP-TR-2002-4055. The histograms for σ_1, ε_1, and E_1 of IM-7/5250-4 composite specimens are shown in Figure 2, together with the fitted normal PDFs. The means and standard deviations of these fitted normal PDFs are listed in Table 1.

The transverse composite specimens were prepared with the configuration of eight-ply rectangular test specimen that was 0.5 inch wide and 10 inches long, and were tested in accordance with ASTM test procedure D3039. There were 61 specimens, with V_f =0.56, tested at room temperature. A FORTRAN computer program was used to detect the outliers of the test results and to calculate the frequencies of the histogram for the remaining 59 specimens. The test data were presented in the Appendix B of AFRL-ML-WP-TR-2002-4055. The histogram for E_2 of IM-7/5250-4 composite specimens is also shown in Figure 2, together with the fitted normal PDFs. The means and standard deviations of these fitted normal PDFs are listed in Table 1.

The 45^o angle-ply composite specimens were prepared with the configuration of eight-ply rectangular test specimen that was 0.065 inch thick and 8 inches long, and were tested in accordance with ASTM test procedure D3518 to measure G_{12} of IM-7/5250-4 composite. There were 57 specimens, with V_f =0.56, tested at room temperature. A FORTRAN computer program was used to detect the outliers of the test results, and to calculate the frequencies of the histogram for the remaining 52 specimens. The test data were presented in the Appendix B of AFRL-ML-WP-TR-2002-4055. The histogram for G_{12} of IM-7/5250-4 composite specimens is shown in Figure 2, together with the fitted normal PDF. The means and standard deviations of these fitted normal PDF are listed in Table 1.

IM-7 /5250-4 Unidirectional Composite

	σ_1 (KSI)	ε_1 (%)	E_1 (MSI)	ν_{12}	E_2 (MSI)	G_1 (MSI)	τ_{12} (KSI)	ν_{21}
Mean (m)	419.22	1.563	24.23	0.313	1.37	0.917	12.3254	0.0177
Standard Deviation (s)	16.05	0.06	0.5	0.015	0.03	0.015	0.3263	0.00085

Table 1. Means and standard deviations for IM-7/5250-4 unidirectional composite

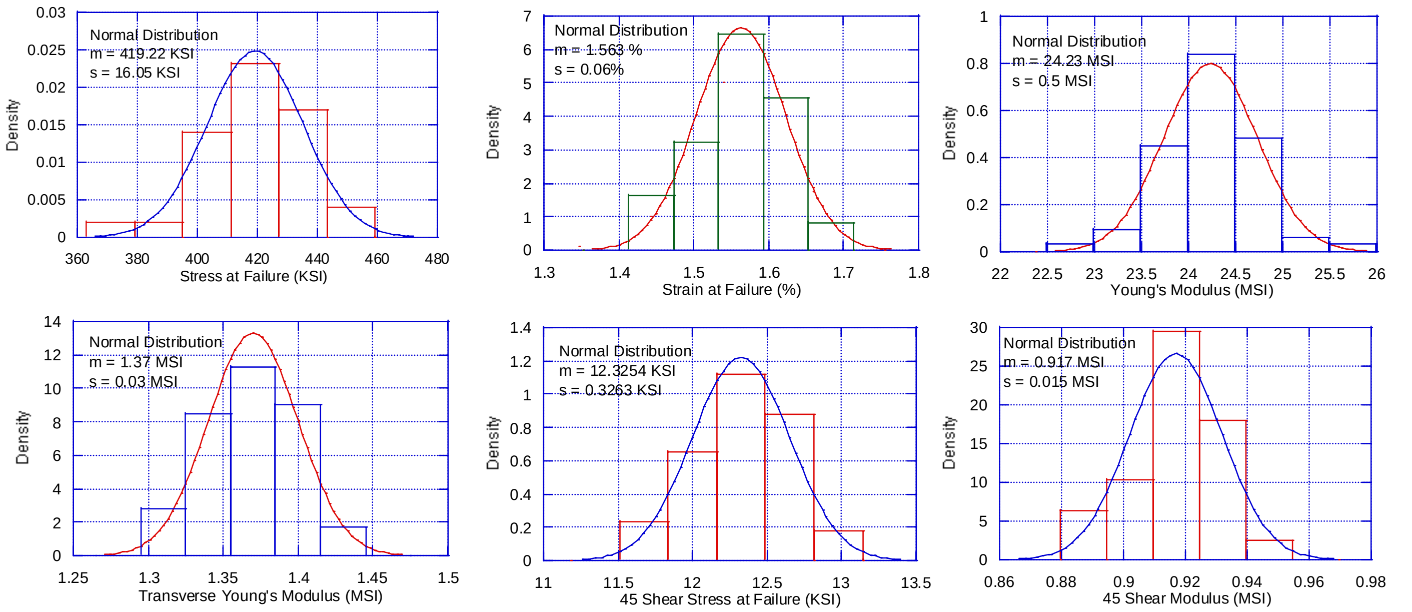

Figure 2. Histograms and Fitted Normal Probability Density Functions for IM-7/5250-4 Unidirectional Fiber-Reinforced Composite Mechanical Properties Test Results

APPLICATION

As shown in Section 3, the observed mechanical properties of IM-7/5250-4 BMI unidirectional lamina fit normal distribution fairly well. In this section, the PDFs for E_1, E_2, ν_{12} and G_{12} of IM-7/5250-4 unidirectional fiber reinforced composite in normal distribution formats are input to the equations in Section 2 to derive the PDFs for the mechanical properties of the IM-7/5250-4 fiber-reinforced composite laminate.

The normal distribution curves and their means and standard deviations for the above PDFs are shown in Figure 2. These PDFs are presented in the normal PDF format.

$$f_x(x) = \frac{1}{\sqrt{2\pi}s_x}\exp\left[-\frac{(x-m_x)^2}{2s_x^2}\right] \qquad (21)$$

The means m and standard deviations s for the above PDFs are listed in Table 1.

In general, Equation (21) with specific mean and standard deviation values for each mechanical property of IM-7/5250-4 unidirectional fiber-reinforced lamina can be input into equations in Section 2 to obtain the predicted PDFs for IM-7/5250-4 fiber-reinforced composite laminate in closed form or in integration formulas. However, there is a simple way to derive these solutions in the case of normal distributions based on the following random variable analyses and theorem [7].

If $X_1, X_2, \ldots\ldots and X_n$ are n normally distributed random variables, then $Y = a_1X_1 + a_2X_2 + \ldots\ldots + a_nX_n$ is normally distributed. Also, if $X_1, X_2, \ldots\ldots and X_n$ are mutually independent random variables, then the mean of the sum is the sum of the means, i.e.,

$$m_y = a_1m_1 + a_2m_2 + \ldots\ldots + a_nm_n \qquad (22)$$

and the variance of the sum is the sum of the variance, i.e.,

$$s_y^2 = a_1^2s_1^2 + a_2^2s_2^2 + \ldots\ldots + a_n^2s_n^2 \qquad (23)$$

Therefore, the PDFs for $X_{ij} = a_{ij}E_1 + b_{ij}E_2 + c_{ij}G_{12}$, ij = 11, 12, 22, 66 in Equations (5, 6, 12, 13, 14) are

$$f_{X_{ij}}\left(X_{ij}\right) = \frac{1}{\sqrt{2\pi}s_X}\exp\left[-\frac{\left(X_{ij}-m_{X_{ij}}\right)^2}{2s_{X_{ij}}^2}\right] \qquad (24)$$

with

$$m_{X_{ij}} = a_{ij} m_{E_1} + b_{ij} m_{E_2} + c_{ij} m_{G_{12}} \tag{25}$$

$$s_{X_{ij}} = \sqrt{\left[a_{ij}^2 s_{E_1}^2 + b_{ij}^2 s_{E_2}^2 + c_{ij}^2 s_{G_{12}}^2\right]} \tag{26}$$

The PDFs for random variables $E_x, E_y or G_{xy}$ in the format of $\frac{1}{X_i} = \frac{a_i}{E_1} + \frac{b_i}{E_2} + \frac{c_i}{G_{12}}$ for i = 1, 2, 4 respectively in Equations (9 and 15) are

$$f_{X_i}(X_i) = \frac{|a_i|}{\sqrt[3]{2\pi} s_{E_1} s_{E_2} s_{G_{12}} X_i^2} \int_0^\infty \int_0^\infty \frac{1}{\left(\frac{1}{X_i} - \frac{b_i}{E_2} - \frac{c_i}{G_{12}}\right)^2} \exp\left(-\frac{\left(\frac{a_i}{\frac{1}{X_i} - \frac{b_i}{E_2} - \frac{c_i}{G_{12}}} - m_{E_1}\right)^2}{2s_{E_1}^2}\right).$$

$$.\exp\left(-\frac{\left(E_2 - m_{E_2}\right)^2}{2s_{E_2}^2}\right) dE_2 \exp\left(-\frac{\left(G_{12} - m_{G_{12}}\right)^2}{2s_{G_{12}}^2}\right) dG_{12} \tag{27}$$

The PDF for random variable ν_{xy} in Equations (9 and 15) is

$$f_{\nu_{xy}}(\nu_{xy}) = \int_0^\infty f_{\frac{\nu_{xy}}{E_x}}\left(\frac{\nu_{xy}}{E_x}\right) f_{E_x}(E_x) \left|\frac{1}{E_x}\right| dE_x \tag{28}$$

where $f_{E_x}(E_x) = \frac{|a_1|}{\sqrt[3]{2\pi} s_{E_1} s_{E_2} s_{G_{12}} E_x^2} \int_0^\infty \int_0^\infty \frac{1}{\left(\frac{1}{E_x} - \frac{b_1}{E_2} - \frac{c_1}{G_{12}}\right)^2} \exp\left(-\frac{\left(\frac{a_1}{\frac{1}{E_x} - \frac{b_1}{E_2} - \frac{c_1}{G_{12}}} - m_{E_1}\right)^2}{2s_{E_1}^2}\right).$

$$.\exp\left(-\frac{\left(E_2 - m_{E_2}\right)^2}{2s_{E_2}^2}\right) dE_2 \exp\left(-\frac{\left(G_{12} - m_{G_{12}}\right)^2}{2s_{G_{12}}^2}\right) dG_{12} \tag{29}$$

and

$$f_{\frac{\nu_{xy}}{E_x}}\left(\frac{\nu_{xy}}{E_x}\right)=\frac{|a_3|}{\sqrt[3]{2\pi}s_{E_1}s_{E_2}s_{G_{12}}}\int_0^\infty\int_0^\infty \frac{1}{\left(\frac{\nu_{xy}}{E_x}-\frac{b_3}{E_2}-\frac{c_3}{G_{12}}\right)^2}\exp\left(-\frac{\left(\frac{a_3}{\frac{\nu_{xy}}{E_x}-\frac{b_3}{E_2}-\frac{c_3}{G_{12}}}-m_{E_1}\right)^2}{2s_{E_1}^2}\right).$$

$$.\exp\left(-\frac{\left(E_2-m_{E_2}\right)^2}{2s_{E_2}^2}\right)dE_2\exp\left(-\frac{\left(G_{12}-m_{G_{12}}\right)^2}{2s_{G_{12}}^2}\right)dG_{12} \qquad (30)$$

MODEL PREDICTION

In this section, the parameters of the fitted normal distributed PDFs for the test data of E_1, E_2, ν_{12} and G_{12} of IM-7/5250-4 unidirectional fiber-reinforced composite, as listed in Table 1, are input into the equations developed in Section 4 to predict the probability density functions for the extensional stiffness A_{ij}, coupling stiffness B_{ij}, bending stiffness D_{ij}, Young's modulus E_x, transverse Young's modulus E_y, shear modulus G_{xy}, and Poisson's ratio ν_{xy} for the regular cross-ply laminate with IM-7/5250-4 fiber-reinforced orthotropic layers.

The probability density functions (PDFs) for extensional stiffness A_{ij} and bending stiffness D_{ij} , ij = 11, 12, 22 and 66, for the 3, 9, 15, 21 and 27-layered symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced composite, and their trend of variations with the number of layers, are predicted from the analysis. Those predicted PDFs for extensional stiffness A_{11} are shown in Figure 3.

For easy reference, the PDFs of these stiffness are rearranged so that the PDFs of all different stiffness of the same laminate thickness are put together on one page. Predicted probability density functions (PDFs) for extensional stiffness A_{ij} and bending stiffness D_{ij}, ij = 11, 12, 22 and 66, for the 15-layered symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced composite, and their trend of variations with the number of layers, are shown in Figure 4.

The probability density functions (PDFs) for the extensional stiffness A_{ij}, coupling stiffness B_{ij} and bending stiffness D_{ij} , ij = 11, 12, 22 and 66, for the 4, 8, 16, 20 and 28-layered anti-symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced composite, and their trend of variations with the number of layers, are predicted from the analysis. Those predicted PDFs for the extensional stiffness A_{11} or A_{22} are shown in Figure 5.

For easy reference, the PDFs of these stiffness are rearranged so that the PDFs of all different stiffness of the same laminate thickness are put together on one page. Predicted probability density functions (PDFs) for extensional stiffness A_{ij}, coupling stiffness B_{ij} and bending stiffness D_{ij}, ij = 11, 12, 22 and 66, for the 16-layered anti-symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced composite, and their trend of variations with the number of layers, are shown in Figure 6.

The PDFs for Young's modulus E_x , transverse Young's modulus E_y , Poisson's ratio ν_{xy} , and shear modulus G_{xy} for the 3, 9, 15, 21 and 27-layered symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced composite are predicted from the analysis. Those predicted PDFs for Young's modulus E_x for the 3, 9, 15, 21 and 27-layered symmetric regular cross-ply laminate are shown in Figure 7. Those predicted PDFs for Young's modulus E_x, transverse Young's modulus E_y, Poisson's ratio ν_{xy} and shear modulus G_{xy} for the 15-layered symmetric regular cross-ply laminate of IM-7/5250-4 fiber-reinforced layers are shown in Figure 8.

Predicted PDFs for Young's modulus E_x, transverse Young's modulus E_y, Poisson's ratio ν_{xy} and shear modulus G_{xy} are the same respectively for all anti-symmetric cross-ply laminate of IM-7/5250-4 fiber-reinforced composite, and are shown in Figure 9.

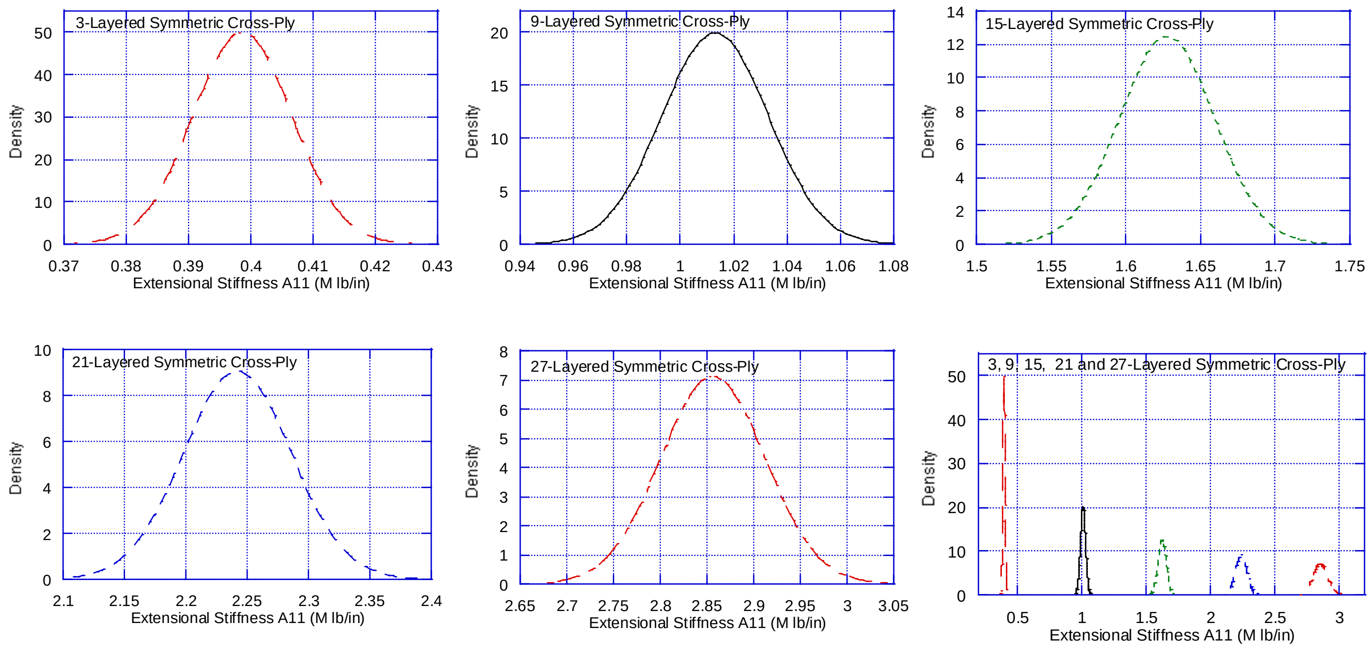

Figure 3. Predicted PDFs for the Extensional Stiffness $A_{11}$of 3, 9, 15, 21 and 27-Layered Symmetric Cross-Ply IM-7/ 5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

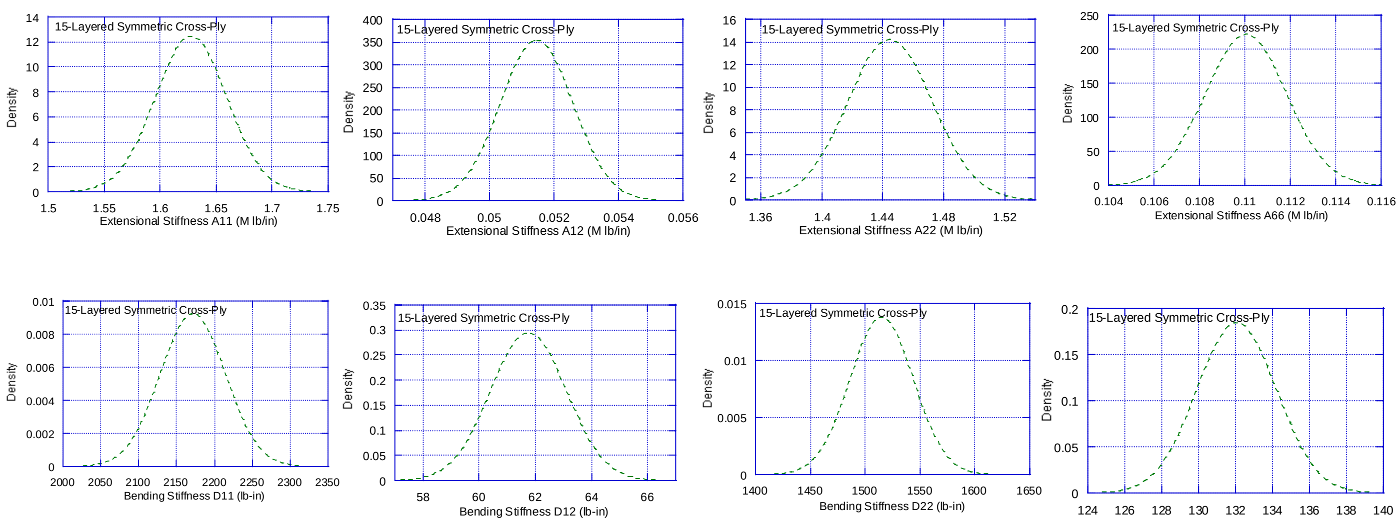

Figure 4. Predicted PDFs for the Extensional Stiffness A_{ij} and Bending Stiffness D_{ij} of 15-Layered Symmetric Cross-Ply IM-7 /5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

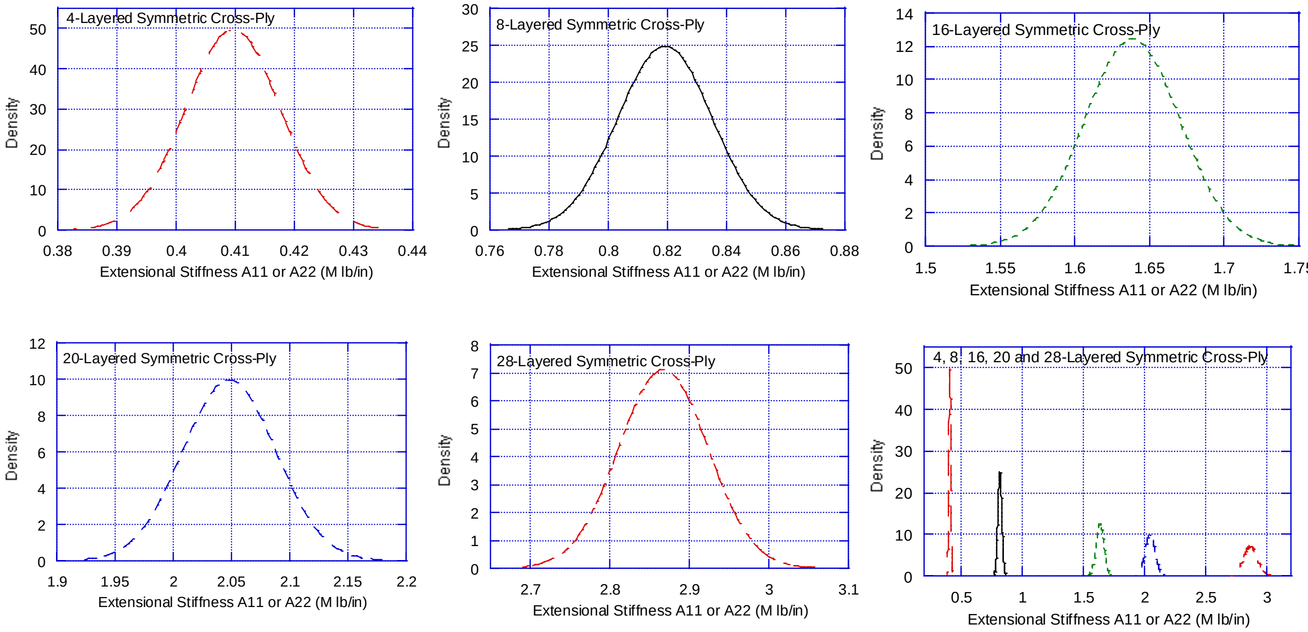

Figure 5. Predicted PDFs for the Extensional Stiffness A_{11} or A_{22} of 4, 8, 16, 20 and 28-Layered Anti-Symmetric Cross-Ply IM-7/5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

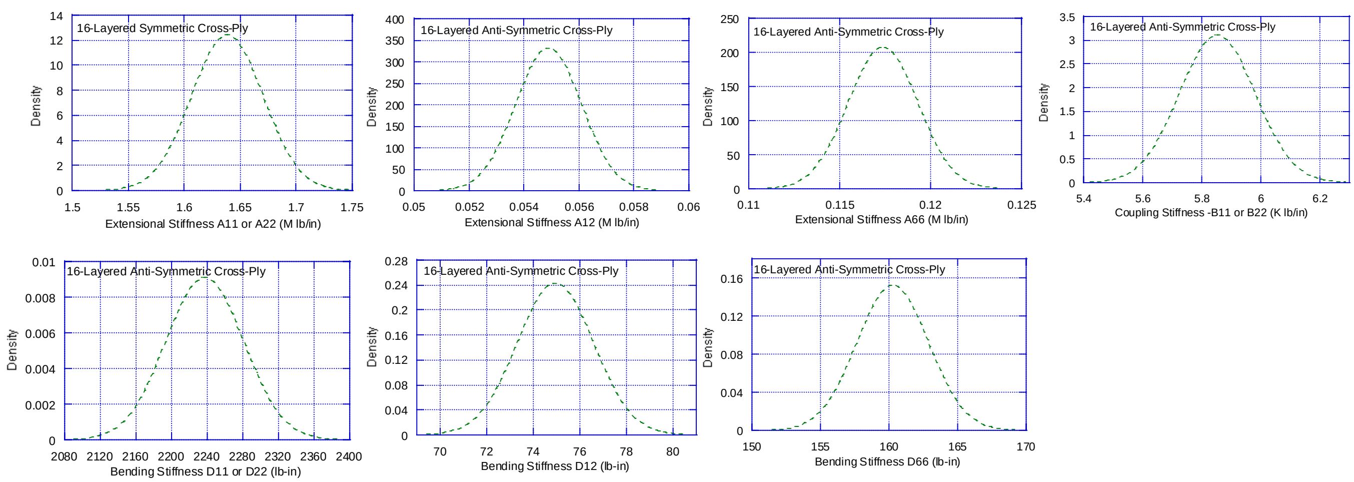

Figure 6. Predicted PDFs for the Extensional Stiffness A_{ij} , Coupling Stiffness B_{ij} and Bending Stiffness D_{ij} of 16-Layered Anti-Symmetric Cross-Ply IM-7/ 5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

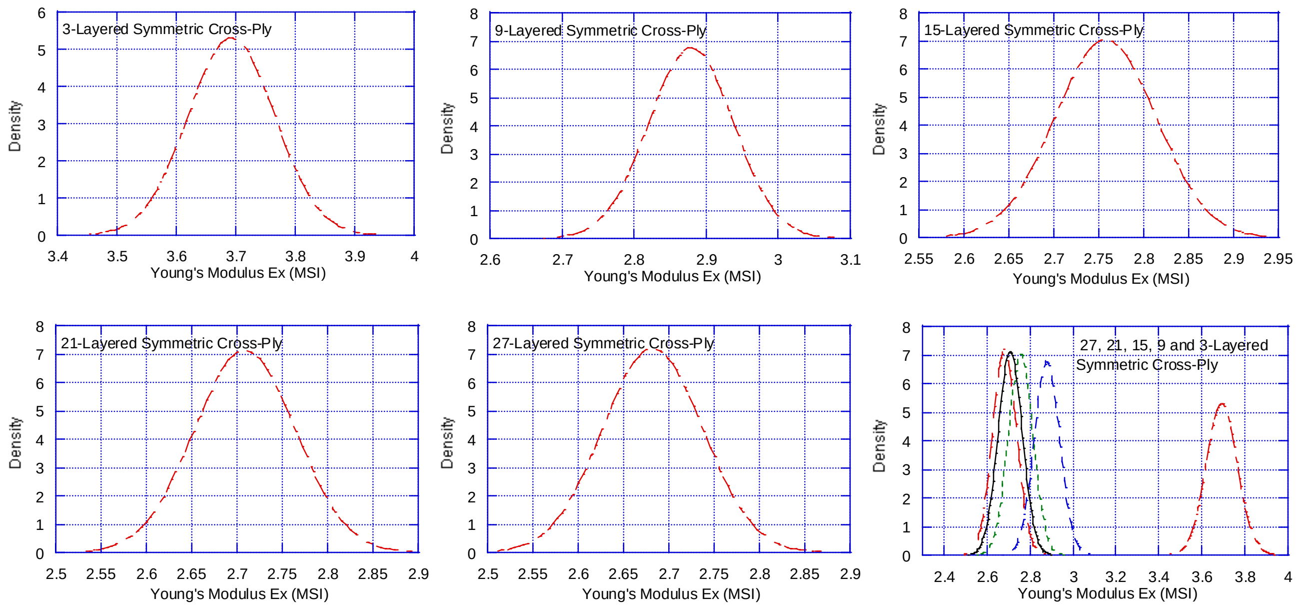

Figure 7. Predicted PDFs for the Young's Modulus E_x of 3, 9, 15, 21 and 27-Layered Symmetric Cross-Ply IM-7/ 5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

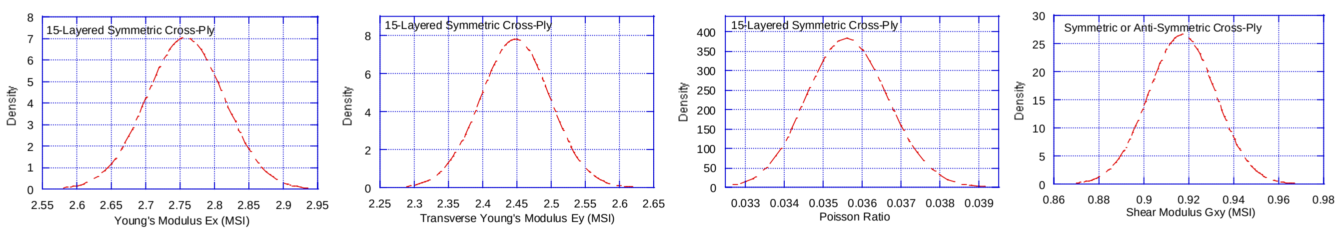

Figure 8. Predicted PDFs for the Young's Modulus E_x, TransverseYoung's Modulus E_y, Poisson's Ratio V_{xy} and Shear Modulus G_{xy} of 15-Layered Symmetric Cross-Ply IM-7/ 5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

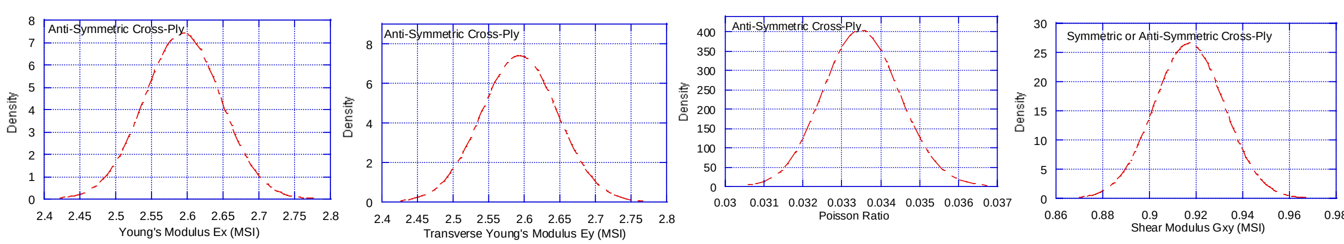

Figure 9. Predicted PDFs for the Young's Modulus E_x, Transverse Young's Modulus E_y, Poisson's Ratio V_{xy} and Shear Modulus G_{xy} of all Anti-Symmetric Cross-Ply IM-7/ 5250-4 Laminated Composite from the Unidirectional Composite with Normally Distributed Mechanical Properties

SUMMARY

Nowadays, the structural design of aerospace structural components using advanced composite materials will include materials design as a part of the structural design process to provide the optimal utilization of the advanced composites in the structural design. Material development is a time consuming and costly task, and new material insertion into production is extremely difficult. To accelerate the insertion of composite materials into AF productions at much lower cost and shorter time, a means must be establish to preclude premature material fabrications and reduce material testing to a minimum but enough to manage the uncertainties in material development.

The angle-ply and cross-ply laminates have been used extensively in aerospace structural designs. It is logical to extend the probabilistic analysis to predict the probability density functions for the extensional, coupling and bending stiffness, Young's modulus, transverse Young's modulus, shear modulus and Poisson's ratio of laminated composites. In which, the probabilistic analysis of the classical lamination theory are utilized to demonstrate the variations of the predicted probability density functions for these mechanical properties of the laminated composites with the numbers and thickness of layers, stacking sequence, and the angles off-axis from the laminate's principal direction. In [3], we had provided a probabilistic analysis on a symmetric angle-ply laminate. In this article, we will focus on the probabilistic analysis for cross-ply laminate of IM-7/5250-4 fiber-reinforced composite with odd-number plies parallel to and even-number plies perpendicular to the laminate principal axes. The probabilistic analysis on anti-symmetric angle-ply composites and composite laminates with various stacking sequences will be discussed in subsequent technical reports.

In this article, a probabilistic laminate mechanics model is developed in Section 2, based on the classical lamination theory and probability and random variable analysis, to predict the probability density functions of these laminate mechanical properties. An AFRL/MLBC in-house test program is established in Section 3 to measure the principal axis mechanical properties of IM-7/5250-4 unidirectional fiber-reinforced composite. The histograms of the test data of the mechanical properties of IM-7/5250-4 composite are fitted very well with normal probability density functions. In Section 4, the assumption of normal probability density functions for the principal axis mechanical properties of IM-7/5250-4 unidirectional composite are used in the development of the probabilistic laminate mechanics model, based on the classical lamination theory, for the probability density functions of the mechanical properties of IM-7/5250-4 cross-ply laminate. In Section5, the numerical values of the fitted normal probability functions for the mechanical properties of IM-7/5250-4 unidirectional composite are inserted into the equations in Section 4 to present graphically the predicted probability density functions for the mechanical properties of the IM-7 /5250-4 cross-ply laminate.

The probabilistic micromechanics model for composite laminates provides a tool for performing preliminary material design analytically as commonly used by the structural designers in the general derivation of probability of failure [4]. The predictions by the probabilistic model will help material producers to eliminate the unnecessary time-consuming and costly material fabrications and to reduce the number of testing, if fabricated, to a minimum but enough to verify the model prediction. And consequently, it provides a means to accelerate the insertion of materials into AF weapon systems

REFERENCES

[1] Chuang S. N., 2004, "The Probabilistic Analysis in Composite Material Design Screening for Unidirectional Fiber-Reinforced Composites", AFRL-ML-WP-TR-2004-4033
[2] Chuang S. N., 2004, "Probabilistic Analysis for the Off-Axis Mechanical Properties of Unidirectional Fiber-Reinforced Composites", AFRL-ML-WP-TR-2004-4259
[3] Chuang S. N., 2004, "Probabilistic Analysis for the Mechanical Properties of Symmetric Angle-Ply Fiber-Reinforced Composite Laminate", AFRL-ML-WP-TR-2005-4099
[4] Gray P. M., and Riskalla M. G., 1997, "Development of Probabilistic Design Methodology for Composite Structures", Vought Aircraft Company Technical Report DOT/FAA/AR-95/17
[5] Tsai S. W., and Hahn H. T., 1980, "Introduction to Composite Materials", TECHNOMIC Publishing Co., Inc
[6] Jones R. M., 1975, "Mechanics of Composite Materials", SCRIPTA Book Company
[7] Soong T. T., 1981, "Probabilistic Modeling and Analysis in Science and Engineering", John Wiley & Sons
[8] Stefansky W., 1972, "Rejecting Outliers in Factorial Designs", Technometrics, Vol 14, 469-479